Skalárszorzat

1.

$\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cdot \cos\gamma$

$-10\sqrt{3} = 5 \cdot |\underline{b}| \cdot \cos 150^\circ$

$|\underline{b}| = 4$

2.

a) $2 \cdot \underline{a} - 3 \cdot \underline{b} = 2 \cdot (3;\ 5) - 3 \cdot (-5;\ 7) = (6;\ 10) - (-15;\ 21) = (21;\ -11)$

b) $|\underline{a} + \underline{b}| = |(-2;12)| = \sqrt{(-2)^2 + 12^2} = \sqrt{148} = 12{,}17$

$\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cdot \cos\gamma$

$\underline{a} \cdot \underline{b} = 3 \cdot (-5) + 5 \cdot 7 = 20$

$|\underline{a}| = \sqrt{3^2 + 5^2} = \sqrt{34} = 5{,}83$

c) $|\underline{b}| = \sqrt{(-5)^2 + 7^2} = \sqrt{74} = 8{,}6$

így :

$20 = \sqrt{34} \cdot \sqrt{74} \cdot \cos\gamma$

$\gamma = 66{,}5^\circ$

3. A megoldás teljesen hasonló az előzőhöz:

a) $3a - 4b = (-22\ ;\ -8)$

b) $|\underline{a} - \underline{b}| = |(-6;-1)| = \sqrt{37}$

c) $\gamma = 65{,}22^\circ$

4. $(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = (10;-4) \cdot (20;20) = 200 - 80 = 120$

5.

$\underline{a} \cdot \underline{b} = 6x + 40 = 112$

$x = 12$

$|\underline{a}| = 13$

$|\underline{b}| = 10$

így :

$112 = 13 \cdot 10 \cdot \cos\gamma$

$\gamma = 30{,}51^\circ$

6.

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$\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cdot \cos\gamma$

$\underline{a} \cdot \underline{b} = 6 \cdot 8 + 8 \cdot (-15) = -72$

$|\underline{a}| = \sqrt{6^2 + 8^2} = 10$

$|\underline{b}| = \sqrt{8^2 + (-15)^2} = 17$

így :

$-72 = 10 \cdot 17 \cdot \cos\gamma$

$\gamma = 115{,}06^\circ$

7.

a) $2\overrightarrow{AB} - 5\overrightarrow{CB} = 2(10;12) - 5(16;-2) = (20;24) - (80;-10) = (-60;34)$

b) $F_{BC} = \left(\frac{12 + (-4)}{2}; \frac{4+6}{2}\right) = (4;5)$

$\gamma = \angle\left(\overrightarrow{CA}; \overrightarrow{CB}\right)$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = \left|\overrightarrow{CA}\right| \cdot \left|\overrightarrow{CB}\right| \cdot \cos\gamma$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = (6;-14) \cdot (16;-2) = 96 + 28 = 124$

c) $\left|\overrightarrow{CA}\right| = \sqrt{36 + 196} = \sqrt{232}$

$\left|\overrightarrow{CB}\right| = \sqrt{256 + 4} = \sqrt{258}$

$124 = \sqrt{232} \cdot \sqrt{258} \cdot \cos\gamma$

$\gamma = 59{,}55^\circ$

Sinus - tétel

8.

$\frac{\sin\gamma}{\sin\beta} = \frac{c}{b} \quad \rightarrow \quad \gamma = 73{,}25^\circ \quad ; \quad \alpha = 180^\circ - (\beta + \gamma) = 56{,}75^\circ$

$\frac{a}{b} = \frac{\sin\alpha}{\sin\beta} \quad \rightarrow \quad a = 8{,}73\ cm$

9.

$\alpha = 55^\circ, \beta = 70^\circ \quad \rightarrow \quad \gamma = 180^\circ - (\alpha + \beta) = 55^\circ$

$\alpha = \gamma \quad \rightarrow \quad BC = AB = 120\,m$

$\sin 35^\circ = \frac{\frac{AC}{2}}{BC} = \frac{AC}{240} \quad \rightarrow \quad AC = 137{,}66\,m$

10.

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$\alpha = 70^\circ, \beta = 50^\circ \quad \rightarrow \quad \gamma = 180^\circ - (\alpha + \beta) = 60^\circ$

$\frac{AC}{AB} = \frac{\sin\beta}{\sin\gamma} \quad \rightarrow \quad AC = 70{,}76\,\text{m}$

$\frac{BC}{AB} = \frac{\sin\alpha}{\sin\gamma} \quad \rightarrow \quad BC = 86{,}81\,\text{m}$

11.

$BAC\angle = DCA\angle = \alpha_1 = 30^\circ \; ; \; DAC\angle = ACB\angle = \alpha_2 = 40^\circ \; ; \; AC = e = 12\,\text{cm}$

$ABC\angle = ADC\angle = \beta = 180^\circ - (\alpha_1 + \alpha_2) = 110^\circ$

$\frac{AB}{AC} = \frac{\sin\alpha_2}{\sin\beta} \quad \rightarrow \quad AB = 8{,}21\,\text{cm}$

$\frac{BC}{AC} = \frac{\sin\alpha_1}{\sin\beta} \quad \rightarrow \quad BC = 6{,}39\,\text{cm}$

$K = 2(AB + BC) = 29{,}2\ \text{cm}$

$T = AB \cdot BC \cdot \sin\alpha = 49{,}3\ \text{cm}^2$

12.

$BAC\angle = \alpha_1 = 35^\circ \; ; \; DAC\angle = \alpha_2 = 45^\circ \; ; \; AC = e = 8\,\text{cm}$

$ABC\angle = BAD\angle = \alpha = 80^\circ \; ; \; BCD\angle = ADC\angle = \beta = 180^\circ - \alpha = 100^\circ$

$BCA\angle = \beta_1 = 180^\circ - (\alpha_1 + \alpha) = 65^\circ$

$\frac{AB}{AC} = \frac{\sin\beta}{\sin\alpha} \quad \rightarrow \quad AB = 7{,}36\ \text{cm}$

$\frac{BC}{AC} = \frac{\sin\alpha_1}{\sin\alpha} \quad \rightarrow \quad BC = 4{,}66\,\text{cm}$

$\frac{CD}{AC} = \frac{\sin\alpha_2}{\sin\beta} \quad \rightarrow \quad CD = 5{,}74\,\text{cm}$

$\sin\alpha = \frac{\text{m}}{BC} \rightarrow \quad \text{m} = 4{,}59\,\text{cm}$

$K = AB + CD + 2BC = 22{,}42\ \text{cm}$

$T = \frac{AB + CD}{2} \cdot \text{m} = 30{,}06\ \text{cm}^2$

13.

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Az átlók metszéspon tja : M

$DM : MB = 2:3 \quad \rightarrow \quad 2 \cdot (2x + 3x) = 80 \quad \rightarrow \quad x = 8 \quad \rightarrow \quad DM = 16, MB = 24$

$BMC\angle = 35^\circ \quad \rightarrow \quad CMD\angle = 145^\circ$

$$\sin \frac{145^\circ}{2} = \frac{\frac{CD}{2}}{DM} \quad \rightarrow \quad CD = 30{,}52$$

$$\sin \frac{145^\circ}{2} = \frac{\frac{AB}{2}}{BM} \quad \rightarrow \quad AB = 45{,}78$$

$BC^2 = MC^2 + MB^2 - 2MC \cdot MB \cdot \cos 35^\circ \quad \rightarrow \quad BC = 14{,}24$

A kisebb alapot a hosszabbra vetítve keletkezik két egyenlő hosszúságú szakasz a hosszabb alap végpontjai n, ezek hossza x

$$x = \frac{AB - CD}{2} = 7{,}63$$

$$m = \sqrt{BC^2 - x^2} = 12{,}02$$

$$K = AB + CD + 2BC = 104{,}78$$

$$T = \frac{AB + CD}{2} \cdot \mathrm{m} = 458{,}56$$

14.

$AB = 24\,cm; CD = 14\,cm; \alpha = 70^\circ; \beta = 50^\circ$

A trapéz CD szárát párhuzamosan eltoljuk A – ig, így kapjuk az AB' D háromszöget.

Ebben a $B'CA\angle = \gamma = 180^\circ - (\alpha + \beta) = 60^\circ$ *és* $AB' = AB - CD = 10\,cm.$

$$\frac{AD}{AB'} = \frac{\sin\beta}{\sin\gamma} \quad \rightarrow \quad AD = 8{,}85\,cm$$

$$\frac{B'D}{AB'} = \frac{\sin\alpha}{\sin\gamma} \quad \rightarrow \quad BC = B'D = 10{,}85\,cm$$

Az átlók :

$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos\beta \quad \rightarrow \quad AC = 18{,}95\,cm$

$BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cdot \cos\alpha \quad \rightarrow \quad BD = 22{,}56\,cm$

15.

A megoldás teljesen hasonló az előző feladatéhoz:

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$AB=15\,cm; CD=10\,cm; \alpha=55^\circ; \beta=75^\circ$

A trapéz BC szárát párhuzamosan eltoljuk D – ig, így kapjuk az AB' D háromszöget.

Ebben a $B'CA\angle=\gamma=180^\circ-(\alpha+\beta)=50^\circ$ *és* $AB'=AB-CD=5\,cm.$

$$\frac{AD}{AB'}=\frac{\sin\beta}{\sin\gamma} \quad\rightarrow\quad AD=6{,}3\,cm$$

$$\frac{B'D}{AB'}=\frac{\sin\alpha}{\sin\gamma} \quad\rightarrow\quad BC=B'D=5{,}35\,cm$$

Az átlók :

$$AC^2=AB^2+BC^2-2AB\cdot BC\cdot\cos\beta \quad\rightarrow\quad AC=14{,}56\,cm$$

$$BD^2=AB^2+AD^2-2AB\cdot AD\cdot\cos\alpha \quad\rightarrow\quad BD=12{,}5\,cm$$

16.

$b=a+1; \alpha=40^\circ; \beta=60^\circ \quad\rightarrow\quad \gamma=80^\circ$

$$\frac{b}{a}=\frac{a+1}{a}=\frac{\sin\beta}{\sin\alpha} \quad\rightarrow\quad a=2{,}88\,cm; b=3{,}88\,cm$$

$$\frac{c}{a}=\frac{\sin\gamma}{\sin\alpha} \quad\rightarrow\quad a=4{,}41\,cm$$

17.

$AC=12\,cm; \alpha:\beta:\gamma:\delta=2:5:6:5 \quad\rightarrow\quad 2x+5x+6x+5x=360^\circ \quad\rightarrow\quad x=20^\circ$

$\alpha=40^\circ; \beta=100^\circ; \gamma=120^\circ; \delta=100^\circ$

A szimmetriaátló felezi a szögeket : $\alpha-t$ *és* $\beta-t$*, így*

$$\frac{AB}{AC}=\frac{\sin 60^\circ}{\sin 100^\circ} \quad\rightarrow\quad AB=AD=10{,}55\,cm$$

$$\frac{BC}{AC}=\frac{\sin 20^\circ}{\sin 100^\circ} \quad\rightarrow\quad BC=CD=4{,}17\,cm$$

18.

$\alpha_1=CAD\angle=56^\circ; \alpha_2=CAB\angle=25^\circ; \gamma_1=DCA\angle=49^\circ; \gamma_2=BCA\angle=23^\circ$

$CDA\angle=180^\circ-(\alpha_1+\gamma_1)=75^\circ; ABC\angle=180^\circ-(\alpha_2+\gamma_2)=132^\circ$

$$\frac{AB}{AC}=\frac{\sin 23^\circ}{\sin 132^\circ} \quad\rightarrow\quad AB=9{,}46\,cm$$

$$\frac{BC}{AC}=\frac{\sin 25^\circ}{\sin 132^\circ} \quad\rightarrow\quad BC=10{,}24\,cm$$

$$\frac{CD}{AC}=\frac{\sin 56^\circ}{\sin 132^\circ} \quad\rightarrow\quad CD=20{,}08\,cm$$

$$\frac{DA}{AC}=\frac{\sin 49^\circ}{\sin 132^\circ} \quad\rightarrow\quad DA=18{,}28\,cm$$

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19.

$$\frac{\sin BF_{AC}C\angle}{\sin\gamma}=\frac{a}{s_b} \quad \rightarrow \quad BF_{AC}C\angle=70{,}53^\circ \ \ vagy \ \ BF_{AC}C\angle=109{,}47^\circ$$

$Az\ előlő\ megoldást\ választva: BF_{AC}C\angle=70{,}53^\circ$, $így\ BF_{AC}A\angle=109{,}47^\circ$

$Ezért\ F_{AC}BC\angle=180^\circ-\left(70{,}53^\circ+45^\circ\right)=64{,}47^\circ$

$$\frac{F_{AC}C}{a}=\frac{\sin 64{,}47^\circ}{\sin 70{,}53^\circ} \quad \rightarrow \quad \frac{b}{2}=F_{AC}C=7{,}66\,cm \quad \rightarrow \quad b=15{,}32\,cm$$

20.

$\alpha:\beta:\gamma=4:5:6 \quad \rightarrow \quad 4x+5x+6x=180^\circ \quad \rightarrow \quad x=12^\circ$

$így\ \alpha=48^\circ\ ;\ \beta=60^\circ\ ;\ \gamma=72^\circ \quad \rightarrow \quad a<b<c$

$$\frac{c}{a}=\frac{\sin\gamma}{\sin\alpha} \quad \rightarrow \quad c=10{,}24\,cm$$

$$s_c^2=a^2+\left(\frac{c}{2}\right)^2-2a\frac{c}{2}\cos\beta \quad \rightarrow \quad s_c=7{,}02\,cm$$

21.

$BAC\angle=\alpha_1=50^\circ\ ;\ DAC\angle=\alpha_2=30^\circ\ ;\ AC=e=10\,\text{cm}$

$ABC\angle=BAD\angle=\alpha=80^\circ\ ;\ BCD\angle=ADC\angle=\beta=180^\circ-\alpha=100^\circ$

$BCA\angle=\beta_1=180^\circ-\left(\alpha_1+\alpha\right)=50^\circ$

$AB=AC=10\,\text{cm}, \text{mert egyenlő szárú}$

$$\frac{BC}{AC}=\frac{\sin\alpha_1}{\sin\alpha} \quad \rightarrow \quad BC=7{,}78\,\text{cm}$$

$$\frac{CD}{AC}=\frac{\sin\alpha_2}{\sin\beta} \quad \rightarrow \quad CD=5{,}08\ \text{cm}$$

$$\sin\alpha=\frac{\text{m}}{BC} \quad \rightarrow \quad \text{m}=7{,}66\,\text{cm}$$

$$K=AB+CD+2BC=30{,}64\ \text{cm}$$

$$T=\frac{AB+CD}{2}\cdot\text{m}=57{,}76\ \text{cm}^2$$

22.

$AC=56\,\text{cm}$

$$\frac{BC}{AC}=\frac{\sin\alpha}{\sin\beta} \quad \rightarrow \quad \alpha=57{,}04^\circ \quad \rightarrow \quad \gamma=180^\circ-\left(\alpha+\beta\right)=52{,}96^\circ$$

$$\frac{AB}{AC}=\frac{\sin\gamma}{\sin\beta} \quad \rightarrow \quad AB=58{,}35\,\text{cm}$$

Cosinus - tétel

23.

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$AB = 12\text{ cm}$, $BC = 8\text{ cm}$, $AC = 16\text{ cm}$

$\mathrm{AC}^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos\beta \quad \rightarrow \quad \beta = \delta = 104{,}48°$

$\dfrac{\sin\dfrac{\alpha}{2}}{\sin\beta} = \dfrac{BC}{AC} \quad \rightarrow \quad \alpha = 57{,}91°$

$\gamma = 360° - (\alpha + 2\beta) = 93{,}13°$

$\sin\dfrac{\alpha}{2} = \dfrac{\dfrac{BD}{2}}{AB} \quad \rightarrow \quad BD = 11{,}62\text{ cm}$

24.

$AB = 3\text{ cm}$, $AC = 6\text{ cm}$, $\alpha_1 = BAC\angle = 40°$

$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos\alpha_1 \quad \rightarrow \quad BC = AD = 4{,}17\text{ cm}$

$\dfrac{\sin\beta}{\sin\alpha_1} = \dfrac{AC}{BC} \quad \rightarrow \quad \beta = 67{,}65°$

$BAD\angle = BCD\angle = \alpha = 180° - \beta = 112{,}35°$

$BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cdot \cos\alpha \quad \rightarrow \quad BD = 5{,}99\text{ cm}$

25.

$AB = CD = 5\text{ cm}$, $BC = AD = 8\text{ cm}$, $AC = 11\text{ cm}$

$\mathrm{AC}^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos\beta \quad \rightarrow \quad \beta = \delta = 113{,}58°$

$BAC\angle = ACD\angle = \alpha_1$

$\dfrac{\sin\alpha_1}{\sin\beta} = \dfrac{BC}{AC} \quad \rightarrow \quad \alpha_1 = 52{,}52°$

$BCA\angle = CAD\angle = \alpha_2 = 180° - (\alpha_1 + \beta) = 13{,}9°$

$\alpha = \alpha_1 + \alpha_2 = 66{,}42°$

$BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cdot \cos\alpha \quad \rightarrow \quad BD = 7{,}55\text{ cm}$

26.

$AB = 64\text{ cm}$, $BC = 90\text{ cm}$, $\beta = \delta = 105°$

$\mathrm{AC}^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos\beta \quad \rightarrow \quad AC = 123{,}2\text{ cm}$

$\dfrac{\sin\dfrac{\alpha}{2}}{\sin\beta} = \dfrac{BC}{AC} \quad \rightarrow \quad \alpha = 89{,}76°$

$\gamma = 360° - (\alpha + 2\beta) = 60{,}24°$

$\sin\dfrac{\alpha}{2} = \dfrac{\dfrac{BD}{2}}{AB} \quad \rightarrow \quad BD = 90{,}32\text{ cm}$

27.

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$BAC\angle = ACD\angle = \alpha_1 = 34{,}2^\circ$; $AB = 30\,\text{cm}$; $AC = e = 22\,\text{cm}$

$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos\alpha_1 \quad \rightarrow \quad BC = AD = 17{,}1\,\text{cm}$

$ABC\angle = BAD\angle = \alpha$

$\frac{AC}{BC} = \frac{\sin\alpha}{\sin\alpha_1} \quad \rightarrow \quad \alpha = 46{,}32^\circ$

$BCD\angle = ADC\angle = \beta = 180^\circ - \alpha = 133{,}68^\circ$

$CAD\angle = \alpha_2 = \alpha - \alpha_1 = 12{,}12°$

$\frac{CD}{AC} = \frac{\sin\alpha_2}{\sin\beta} \quad \rightarrow \quad CD = 6{,}39\,\text{cm}$

28.

$a = 9$ cm , $b = 15$ cm , $s_b = 12$ cm

$s_b{}^2 = a^2 + \left(\frac{b}{2}\right)^2 - 2a \cdot \left(\frac{b}{2}\right) \cdot \cos\gamma \quad \rightarrow \quad \gamma = 92{,}87°$

$c^2 = a^2 + b^2 - 2a \cdot b \cdot \cos\gamma \quad \rightarrow \quad c = 17{,}87$ cm

29.

$AB = a = 12$ cm; $CD = c = 6$ cm; $BC = b = 4$ cm; $DA = d = 5$ cm;

A trapéz BC szárát párhuzamosan eltoljuk D – ig, így kapjuk az AB' D háromszöget.

Ebben a $AB' = 6$ cm; $B'D = 4$ cm; $DA = 5$ cm; $ADB'\angle = \varphi$

$AB'^2 = DA^2 + DB'^2 - 2DA \cdot DB' \cdot \cos\varphi \rightarrow \quad \varphi = 82{,}82°$

$\frac{DB'}{AB'} = \frac{\sin\alpha}{\sin\varphi} \quad \rightarrow \quad \alpha = 41{,}41° \quad \rightarrow \quad \beta = 180° - (\alpha + \varphi) = 55{,}77°$

így

$\alpha = DAB'\angle = DAB\angle = 41{,}41°$; $\beta = AB'D\angle = ABC\angle = 55{,}77°$

$\gamma = 180° - \beta = 124{,}23°$; $\delta = 180° - \alpha = 138{,}59°$

$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos\beta \quad \rightarrow \quad AC = 10{,}3\,\text{cm}$

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